

# Entanglement distillation in optomechanics via unsharp measurements

Víctor Montenegro,<sup>1,\*</sup> Alessandro Ferraro,<sup>2,†</sup> and Sougato Bose<sup>1,‡</sup>

<sup>1</sup>*Department of Physics and Astronomy, University College London,  
Gower Street, London WC1E 6BT, United Kingdom*

<sup>2</sup>*Centre for Theoretical Atomic, Molecular, and Optical Physics,  
School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom*

(Dated: March 17, 2015)

Quantum technologies based on optical Gaussian states have proven very promising in terms of scalability. However, their use in quantum networking is hindered by the fact that Gaussian entanglement cannot be distilled via Gaussian operations. We take advantage of hybrid optomechanical systems to address this problem, proposing a scheme to distill optical two-mode squeezed vacua via unsharp measurements. Here, one of the optical modes is injected into a single sided Fabry-Pérot cavity and non-linearly coupled to a mechanical oscillator. Afterwards, the position of the oscillator is measured using pulsed optomechanics and homodyne detection. Our results show that this measurement can conditionally increase the initial entanglement under an optimal radiation-pressure interaction strength, which corresponds to an effective unsharp non-Gaussian measurement of the photon number inside the cavity. We show how the resulting entanglement distillation can be verified by using a standard quantum teleportation procedure.

**Introduction.**— Recent experiments with quantum optics have demonstrated the generation of entanglement across hundreds of modes and partitions [1–3], thus offering unprecedented opportunities for quantum networking [4]. However, the states generated in these systems (Gaussian states of light) suffer from the drawback that entanglement distillation — a pivotal primitive for long distance quantum communication and networking [5] — is not readily available. This is due to the fact that the interactions naturally occurring in these systems are Gaussian and a “no-go theorem” prevents Gaussian operations to distill Gaussian entanglement [6]. In order to overcome this roadblock, purely optical methods involving non-Gaussian operations have been suggested [7–10], with the dominant scheme relying on photon subtraction [11, 12]. The implementation of such schemes is currently topical but remains technologically challenging [13–18]. We introduce here an alternative route based on hybrid opto-mechanical systems that naturally possess non-Gaussian radiation-pressure interactions.

Quantum optomechanics is opening up new avenues for the manipulation of optical states [19, 20]. The usage of optomechanical systems for teleportation and establishing Gaussian entangled states of distant systems have been studied (see, *e.g.* Refs. [21–24]). However the key quantum communication enabling protocol of entanglement distillation has thus far been untouched in opto-mechanics as the majority of the applications considered a linearized (therefore Gaussian) interaction. On the other hand, the bare optomechanical radiation pressure interaction is non-Gaussian (tri-linear) [25]. Can this coupling be useful for quantum communication, namely, for entanglement distillation? The trilinear coupling, which enables mechanical superpositions [26–28], has only recently started drawing serious attention [29–37] as it is becoming physically signif-

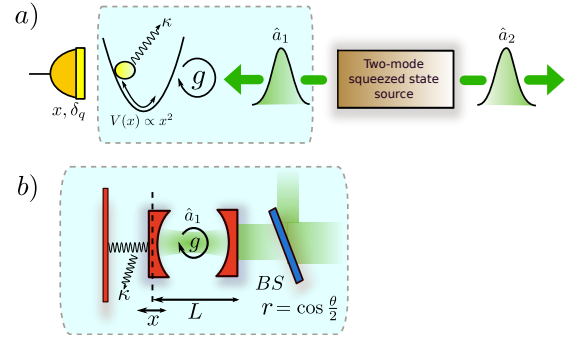


FIG. 1: (Color online) Distillation procedure for two-mode squeezed vacuum (TMSV). In a) we show the general scheme, where a source generates a TMSV, the first mode  $\hat{a}_1$  interacts with a damped ( $\kappa$ ) mechanical harmonic oscillator  $V(x) \propto x^2$  ( $g$  is the scaled coupling strength). After this interaction, and conditioned on unsharp measurements of the position  $x$  of the mechanical oscillator with resolution  $\delta_q$ , the initial TMSV entanglement between  $\hat{a}_1$  and  $\hat{a}_2$  can be increased. In b) we show the explicit realization of the same procedure: here we substituted the oscillator with a single sided Fabry-Pérot cavity with a movable mirror, modeling the injection of the mode  $\hat{a}_1$  into the cavity via BS of reflectivity  $r$ . The position of the mirror is to be measured by pulsed optomechanics [46].

icant in certain setups [38–43]. Here we show that the bare “tri-linear” optomechanical radiation pressure interaction can enable the concentration of the entanglement of two mode squeezed vacua by local operations. In particular, “snap-shot” position detections of a mechanical oscillator, whose technology has been developed recently [44–46], serve as the alternative to photo-detection. Our proposal also demonstrates that sometimes the usage of weak (in the sense of “coarse-grained”/unsharp) measurements can be better for enacting a quantum protocol than their fine grained counterparts.

**System dynamics.**— Let us commence by considering

two light-modes (with corresponding annihilation operators  $\hat{a}_1$  and  $\hat{a}_2$  satisfying  $[\hat{a}_j, \hat{a}_j^\dagger] = 1$  for  $j = 1, 2$ ) in a two-mode squeezed vacuum (TMSV)  $|\psi(0)\rangle_{\text{TMSV}} = \sqrt{1-\lambda^2} \sum_{n=0}^{\infty} \lambda^n |n, n\rangle_{12}$ , with  $\lambda = \tanh s$  and  $s$  being the squeezing parameter. One light beam ( $\hat{a}_1$ ) is coupled to a mechanical harmonic oscillator ( $m$ ), whereas mode  $\hat{a}_2$  propagates freely [a general scheme is illustrated in Fig. (1-a)]. As said, we focus our attention on a Fabry-Pérot configuration [see Fig. (1-b)] where the mode  $\hat{a}_1$  is injected into a cavity. Such injection of a propagating optical mode into a cavity is standard in LIGO [47], and is considered standard in the study of cavity-based quantum networks [23, 45, 48]. Obviously, the injection itself entails a decoherence of the optical field, which we will duly take into account.

After mode  $\hat{a}_1$  is injected into the cavity it starts interacting with the mechanical oscillator via an optomechanical Hamiltonian. In a frame rotating at the frequency  $\omega_1$  of mode  $\hat{a}_1$ , this is given by  $\hat{H}_{\text{int}} = \hat{b}^\dagger \hat{b} - g \hat{a}_1^\dagger \hat{a}_1 (\hat{b}^\dagger + \hat{b})$ , where  $g = g_0/\omega_m$  is the scaled coupling parameter,  $\omega_m$  is the angular frequency of the mechanical oscillator ( $\hat{b}$ ),  $g_0 = x_{\text{zpf}} \omega_1/L$  is the radiation-pressure interaction strength,  $L$  is the cavity length at equilibrium, and  $x_{\text{zpf}}$  is the zero-point fluctuation amplitude (we set  $\hbar = 1$ ) [26]. Given the recent possibilities of ground state cooling [23, 40, 49–51] we will assume that the oscillator is initially in a coherent state  $|\alpha\rangle_m$ . The evolution in the absence of any source of decoherence can be solved straightforwardly [52, 53]. In this ideal case, the dynamics is characterized by a displacement of the mirror position, conditioned on the photon number  $n$ :  $\lambda^n |n\rangle_1 |\alpha\rangle_m \rightarrow \lambda^n \exp(ig^2 n^2 (t - \sin t)) \exp(ign \text{Im}[\alpha\eta]) |n\rangle_1 |\alpha e^{-it} + gn\eta\rangle_m$ . Here  $|n\rangle$  is a photonic Fock state,  $\eta = 1 - e^{-it}$ , and  $t$  represents a scaled time, being the actual time multiplied by  $\omega_m$ .

However, in realistic conditions the state will be affected by decoherence. In order to give a full analytic solution we assume that the cavity decay  $\kappa_c/\omega_m \ll g$  (the resolved side-band regime already attained [54], and a goal of several setups aiming to achieve ground state cooling). We solve the standard Markovian master equation at zero temperature for the decoherence of the oscillator following the procedure in the appendix of Ref. [52]. In this case, the master equation reads as:  $d\hat{\rho}(t)/dt = -i[\hat{H}_{\text{int}}, \hat{\rho}(t)] + (2\hat{b}\hat{\rho}(t)\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho}(t) - \hat{\rho}(t)\hat{b}^\dagger\hat{b})\kappa/2$ , being  $\kappa$  the mechanical energy damping rate. Another inevitable source of decoherence is the attenuation due to the injection of the light beam into the cavity. To model this, we consider a beam splitter (BS) in front of the fixed cavity-mirror, such that one port of the latter is fed with mode  $\hat{a}_1$  and the other with a vacuum field [55]. Under these sources of decoherence, the analytic solution for the full

density matrix is

$$\begin{aligned} \hat{\rho}(t) &= |1 - \lambda^2| \sum_{n,m=0}^{\infty} \mathcal{C}_{nm} e^{-D_{nm}^\kappa(t)} \\ &\times \sum_{k=0}^{\min[n,m]} G_{nm}^k(\theta) |n-k, n\rangle_{12} \langle m-k, m|_{12} \\ &\otimes |\phi_n(\kappa, t)\rangle_m \langle \phi_m(\kappa, t)|_m \end{aligned} \quad (1)$$

where the  $\theta$  angle is related with the reflection coefficient of the BS as  $r = \cos(\theta/2)$ . The other terms are

$$\begin{aligned} \mathcal{C}_{nm} &= \lambda^{n+m} e^{ig^2[t - \sin t](n^2 - m^2)} e^{ig \text{Im}[\alpha\eta](n-m)} \\ G_{nm}^k(\theta) &= \sqrt{\binom{n}{k} \binom{m}{k}} \cos^{2k} \frac{\theta}{2} \sin^{n-k} \frac{\theta}{2} \sin^{m-k} \frac{\theta}{2} \\ \phi_n(\kappa, t) &= \frac{ign(1 - e^{-(i+\kappa/2)t})}{i + \kappa/2} + \alpha e^{-(i+\kappa/2)t} \\ D_{nm}^\kappa(t) &= -\frac{\kappa}{2} \int_0^t (|\phi_n(\kappa, t')|^2 + |\phi_m(\kappa, t')|^2 \\ &\quad - 2\phi_n^*(\kappa, t')\phi_m(\kappa, t')) dt'. \end{aligned} \quad (2)$$

*Entanglement distillation.*—To distill the initial TMSV we proceed to measure the quadrature position of the oscillator [46] through an inefficient detector. This corresponds to the positive-operator valued measure (POVM)  $\tilde{\Pi}(q) = 1/\sqrt{2\pi\delta_q^2} \int_{-\infty}^{\infty} \exp(-(q - y)^2/2\delta_q^2) |y\rangle \langle y| dy$ , where  $q = x\sqrt{m\omega_m/\hbar}$  is the dimensionless position of the oscillator (with actual position  $x$ ),  $m$  is the oscillator mass, and  $\delta_q$  determines the measurement resolution [56]. The state (unnormalized) after the measurement, conditioned to an outcome  $q$ , is given by

$$\begin{aligned} \hat{\rho}(t)_{12} &= \frac{|1 - \lambda^2|}{\sqrt{2\pi\delta_q^2}} \sum_{n,m=0}^{\infty} \mathcal{C}_{nm} e^{-D_{nm}^\kappa(t)} \mathcal{I}_{nm} \sum_{k=0}^{\min[n,m]} \\ &\times G_{nm}^k(\theta) |n-k, n\rangle_{12} \langle m-k, m|_{12} \end{aligned} \quad (3)$$

where  $\mathcal{I}_{nm} = \int_{-\infty}^{\infty} \psi_{\phi_n(\kappa, t)}(x) \psi_{\phi_m(\kappa, t)}^*(x) e^{-\frac{(q-x)^2}{2\delta_q^2}} dx$ , in which  $\psi_\xi(q) \equiv \langle q|\xi\rangle$  is the position wave-function of an arbitrary coherent state  $|\xi\rangle$ . The probability density function (PDF) of the outcome  $q$  is

$$p(q) = \frac{|1 - \lambda^2|}{\sqrt{\pi(1 + 2\delta_q^2)}} \sum_{l=0}^{\infty} \lambda^{2l} \exp \left[ \frac{(q - \sqrt{2} \mathbb{R}e[\phi_l(\kappa, t)])^2}{-(1 + 2\delta_q^2)} \right]. \quad (4)$$

To quantify the entanglement we use the negativity [57–59], defined as  $N(t) = 1/2 \sum_i (|\varepsilon_i| - \varepsilon_i)$ , where  $\varepsilon_i$  are the eigenvalues of the partial transposition of the normalized version of  $\hat{\rho}(t)_{12}$  of Eq. (3).

A quick inspection of Eq. (4) reveals that a change in the initial amplitude from  $|\alpha|e^{i\phi_\alpha}$  to  $|\alpha'|e^{i\phi'_\alpha}$  entails a rigid shift of the outcome probability  $p(q)$  by  $\Delta q \approx$

$|\alpha| \cos(\phi_\alpha - t) - |\alpha'| \cos(\phi'_\alpha - t)$ . We verified numerically that also the entanglement negativity is subjected to the same shift, which implies that a change in  $\alpha$  can be accounted for by selecting the measurement outcome  $q$  accordingly. Given this, we set for the rest of this work the initial coherent state to  $\alpha = 0$ .

We now have the ingredients to assess the validity of the distillation procedure. For a fixed set of values ( $\kappa = 0.01, \delta_q \approx 0.11, r = 0.1, t = \pi, \lambda = 0.3$ ) we plot in the left  $y$ -axis of Fig. (2-a) the ratio of the negativity  $N_D/N_0$  (solid line) as a function of the outcome  $q$  of the measurement of the oscillator position, where  $N_D(N_0)$  stands for the distilled (initial) negativity. In the right  $y$ -axis, we show its corresponding PDF (dashed line) as a function of  $q$ . The success probability of the distillation protocol, namely the probability of obtaining  $N_D > N_0$ , is given by the shaded region and is defined as:

$$\Pr(g, \lambda)_s = \int_{N_D > N_0} p(q) dq, \quad (5)$$

In Fig. (2-a) we illustrate three representative cases. For weak optomechanical coupling ( $g = 0.01$ ), one achieves a large success probability though at the cost of an almost negligible increase in negativity  $N_D \approx N_0$ . For intermediate coupling ( $g = 0.2$ ) the negativity is significantly enhanced, still retaining a high success probability. On the other hand, for large coupling ( $g = 1$ ), not only  $N_D \lesssim N_0$  but also the success probability is considerably small.

As a consequence, we see that an optimal region of the coupling value emerges, given that the entanglement distillation is predominantly achieved for *intermediate* radiation-pressure coupling. Similarly we can also see that entanglement distillation is achieved for intermediate values of the initial entanglement, implying that distillation is optimal in the parameter region  $0.2 < \{g, \lambda\} < 0.4$ , for which  $\Pr(g, \lambda)_s > 0.15$  and  $N_D > N_0$ .

The reason for this behaviour can be intuitively understood considering the structure of the TMSV state and its evolution under the distillation protocol. The states of the whole system (in absence of decoherence) before and after the optomechanical interaction are given by  $|0\rangle_m \sum_n \lambda^n |n, n\rangle_{1,2}$  and  $\sum_n \lambda^n e^{ig^2 n^2 \pi} |2gn\rangle_m |n, n\rangle_{1,2}$  respectively. The states  $|2gn\rangle_m$  become more and more distinguishable for larger  $g$ . As a consequence, the measurement of the oscillator position effectively becomes a sharp measurement of Fock state inside the cavity [60] that projects the two light beams into a factorized state  $|n, n\rangle_{1,2}$ . This intuitively explains the failure of the distillation protocol for large  $g$ . The failure for large  $\lambda$  is instead due to the fact that the number of photon Fock states compatible with a specific outcome  $q$  is finite (for any non-zero  $g$ ). For large enough  $\lambda$ , this finite superposition of a small set of Fock states  $|n, n\rangle_{1,2}$  is not enough to exceed the entanglement of the initial TMSV. As said, we are neglecting here the cavity decay and further losses

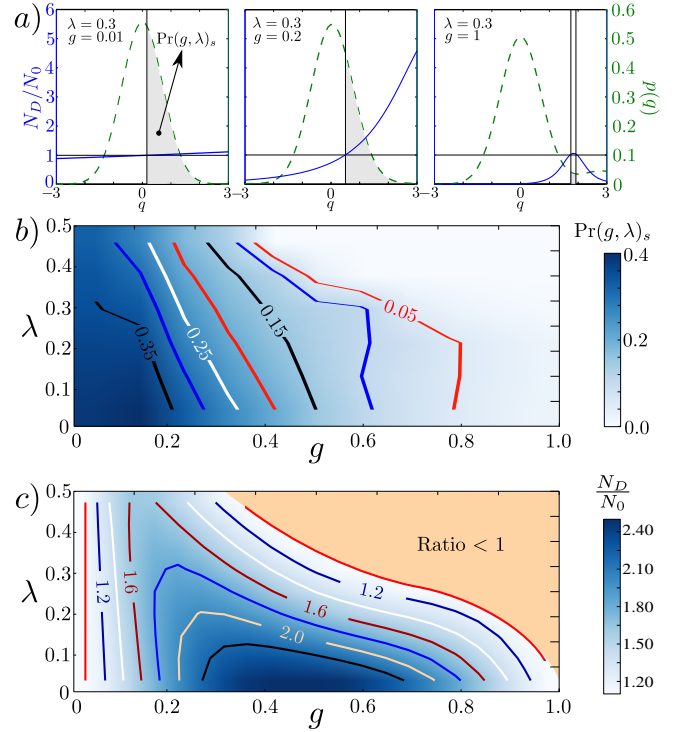


FIG. 2: (Color online) Upper panel a): We plot in the left  $y$ -axis the ratio of the negativity  $N_D/N_0$  as a function of the oscillator's position  $q$  (solid line), where  $N_D$  ( $N_0$ ) stands for the distilled (initial) negativity. In the right  $y$ -axis we show the PDF as a function of  $q$  (dashed line). In the middle panel b) we illustrate the distillation success probability ( $\Pr(g, \lambda)_s$ ) corresponding to the shaded region in the upper panel. Finally, in the bottom panel c) we show the ratio of negativity as function of  $\lambda$  and  $g$  for a specific oscillator's position  $q = 1.5$ .

in the extraction of the distilled state from the cavity. However, let us note that our results indicate that the distillation protocol is robust against large injection losses (in Fig. 2 we considered a beam-splitter reflectivity of  $r = 0.1$ ), which in turn suggests robustness against cavity and extraction losses as well.

*Quantum teleportation with the distilled state.*—It is of course, crucial to suggest both a method to verify the successful distillation of entanglement, as well as an application of the distilled state. In the following we will show how the teleportation of an arbitrary coherent state  $|\beta\rangle$  by the distilled state can serve both purposes. Following the standard procedure [61], we combine mode  $\hat{a}_1$  with the coherent state to teleport into a balanced beam splitter. Subsequently, we measure the position (momentum) quadrature of the transmitted (reflected) beam. The unnormalized state after the joint  $\{\bar{x}, \bar{p}\}$ -measurements cor-

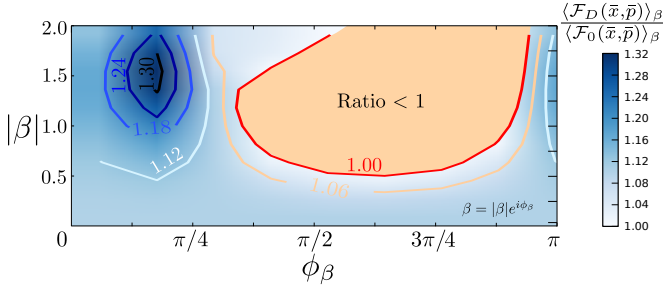


FIG. 3: The figure shows the ratio of the average fidelity ( $\langle \mathcal{F}_D(\bar{x}, \bar{p}) \rangle_\beta / \langle \mathcal{F}_0(\bar{x}, \bar{p}) \rangle_\beta$ ) as a function of the amplitude ( $|\beta|$ ) and phase ( $\phi_\beta$ ) of the coherent state to teleport. In this figure we used the parameters  $\alpha = 2e^{i\pi/4}$ ,  $q = 0$ ,  $\lambda = 0.3$ ,  $g = 0.2$ .

responds to

$$\begin{aligned} \hat{\rho}(t)_2 = & \frac{|1 - \lambda^2|e^{-(\bar{x}^2 + \bar{p}^2) - |\beta|^2}}{\pi p(q)\sqrt{2\pi\delta_q^2}} \sum_{n,m=0}^{\infty} \frac{\mathcal{C}_{nm}}{2^{n+m}} e^{-D_{nm}^\kappa(t)} \\ & \times \mathcal{I}_{nm} \sum_{n',m'=0}^{\infty} \frac{\beta^{n'}\beta^{*m'}}{2^{n'+m'}} \frac{1}{n'!m'!} \sum_{k=0}^{\min[n,m]} G_{nm}^k(\theta) \\ & \times \frac{4^k}{\sqrt{(n-k)!(m-k)!}} \sum_{j,j'=0}^{n-k,n'} \mathcal{D}(n-k, j, n', j') \\ & \times \sum_{l,l'=0}^{m-k,m'} \mathcal{D}^*(m-k, l, m', l') |n\rangle_2 \langle m|_2 \end{aligned} \quad (6)$$

where  $\mathcal{D}(n-k, j, n', j') = \binom{n-k}{j} \binom{n'}{j'} e^{i\frac{\pi}{2}(2j'-n')} H_{j+j'}(\bar{x}) \times H_{n-k+n'-j-j'}(\bar{p})$ , in which  $H_n$  the Hermite polynomial of degree  $n$ . The normalization of Eq. (6) gives us the probability of the joint measurement  $p(\bar{x}, \bar{p}) = \text{Tr} \hat{\rho}(t)_2$ . The success of the teleportation can be quantified by the fidelity which in our case reads as  $f(\bar{x}, \bar{p}) = \langle \beta | \hat{\rho}_2 | \beta \rangle$ , being  $\hat{\rho}_2$  the normalized state appearing in Eq. (6) displaced by  $\hat{D}(\sqrt{2}(\bar{x} + i\bar{p}))$ . In particular, we consider the fidelity averaged over all possible outcomes:

$$\langle \mathcal{F}(\bar{x}, \bar{p}) \rangle_\beta = \int_{-\infty}^{+\infty} f(\bar{x}, \bar{p})_\beta p(\bar{x}, \bar{p})_\beta d\bar{x} d\bar{p}. \quad (7)$$

In Fig. (3), with parameters  $\alpha = 2e^{i\pi/4}$  and  $q = 0$ , we clearly distinguish many instances in which the fidelity with the distilled state is greater than the one with the original state, *i.e.*,  $\langle \mathcal{F}_D(\bar{x}, \bar{p}) \rangle_\beta > \langle \mathcal{F}_0(\bar{x}, \bar{p}) \rangle_\beta$  [the subindex in  $\mathcal{F}_D$  ( $\mathcal{F}_0$ ) corresponds to the distilled (initial) state]. Thus, for a wide range of  $\beta$ , the distilled state can be exploited to improve teleportation fidelity.

*Experimental feasibility.*— The generation of the TMSV state is routine in quantum optics laboratories where  $0 < \lambda < 0.5$  can be fully realized. On the other hand, our protocol requires the initialization of the mechanical oscillator in its ground state or in a coherent state. The cooling of a mechanical oscillator to its quan-

tum ground state has been recently achieved [40, 51]. Finally, typical values in optomechanics show a wide range for the oscillator's damping rate  $\kappa$  [39, 41, 62] compatible with our assumptions. Moreover, our optimal  $g$  value of  $\sim 0.2$  for the optomechanical interaction can now be achieved in several setups [38–43].

The key stage of this work consists in measuring the oscillator. After the pulse  $\hat{a}_1$  interacts with the oscillator under sufficiently weak coupling (e.g., using  $g \approx 0.2$ ,  $\omega_m/2\pi = 500$  kHz, and  $g_0 \simeq 86$  kHz, as in [46]), a second auxiliary pulse with a duration much smaller than the mechanical period is then injected into the cavity. The optical phase of the emerging field (correlated with the mechanical position) is then measured via balanced homodyne detection [46]. Moreover, ideally, we want our optomechanical coupling to cease after the measurement as we do not want entanglement of the distilled quantum state with mechanics while it is being used for some other protocol. This can be achieved, for example, by using trapped mechanical objects as oscillators interacting with a cavity field: an optically trapped object can be pushed away by a strong pulse [63]; a charged object held in a Paul trap can be suddenly shifted relative to the cavity field by suddenly shifting the trap centre [64].

*Concluding remarks.*— We have presented a first application of the bare radiation-pressure coupling in a practical quantum communication protocol, showing how optomechanics can fill a crucial gap in enabling long distance quantum networks using Gaussian states of light. Our proposal uses an indirect measurement of the photon number of the field inside a cavity through the position measurement of a mechanical element coupled to it. For an optimal strength of the coupling, the photon number is measured weakly or “unsharply” and this results in entanglement distillation conditioned to the position outcome. For a vacuum state inside the cavity, the position meter does not move, corresponding to a failed outcome of distillation. Thus our procedure has a degree of similarity with the known purely optical procedure of photon subtraction [7–12], where also the vacuum component is filtered, and we have comparable figures of merit. On the other hand, the position measurements of a mechanical oscillator can be highly precise, especially as we are not concerned about back-reaction as the measurement is at the end of our protocol (the oscillator can be reinitialized before performing distillation again). Optimizations with multiple modes in the cavity can be attempted.

The state obtained through our protocol is non-Gaussian, and thus it can serve as the first step of Gaussianification [9, 65] — which can enhance its entanglement further and act on multiple copies — or, more in general, for quantum computation purposes [66]. Moreover, the procedure here outlined could be useful also in a quantum repeater scenario for long distance communication, considering that there no further extraction of the distilled state from the optical cavity is needed.

V.M. is supported by the Comisión Nacional de Investigación Científica y Tecnológica (Becas Chile Grant No. 72110207). A.F. acknowledges funding from the John Templeton Foundation (Grant No. 43467). S.B. acknowledges the EPSRC Grant No. EP/J014664/1. We have benefited from comments of M. Vanner, G. J. Milburn, N. Kiesel, P. F. Barker and D. E. Browne.

---

\* Electronic address: v.montenegro.11@ucl.ac.uk

† Electronic address: a.ferraro@qub.ac.uk

‡ Electronic address: sougato@theory.phys.ucl.ac.uk

- [1] S. Yokoyama, R. Ukai, S. C. Armstrong, C. Sornphiphatphong, T. Kaji, S. Suzuki, Jun-ichi Yoshikawa, H. Yonezawa, N. C. Menicucci, and A. Furusawa, *Nature Photonics* **7**, 982 (2013).
- [2] M. Chen, N. C. Menicucci, and O. Pfister, *Phys. Rev. Lett.* **112**, 120505 (2014).
- [3] S. Gerke, J. Sperling, W. Vogel, Y. Cai, J. Roslund, N. Treps, and C. Fabre, *Phys. Rev. Lett.* **114**, 050501 (2015).
- [4] H. J. Kimble, *Nature* **453**, 1023 (2008).
- [5] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, *Phys. Rev. A* **53**, 2046 (1996); C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, *Phys. Rev. Lett.* **76**, 722 (1996).
- [6] J. Eisert, S. Scheel, and M. B. Plenio, *Phys. Rev. Lett.* **89**, 137903 (2002); J. Fiurášek, *Phys. Rev. Lett.* **89**, 137904 (2002); G. Giedke and J. I. Cirac, *Phys. Rev. A* **66**, 032316 (2002).
- [7] L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **84**, 4002 (2000).
- [8] J. Fiurášek, L. Mišta Jr., and R. Filip, *Phys. Rev. A* **67**, 022304 (2003).
- [9] D. E. Browne, J. Eisert, S. Scheel, and M. B. Plenio, *Phys. Rev. A* **67**, 062320 (2003).
- [10] A. Datta, L. Zhang, J. Nunn, N. K. Langford, A. Feito, M. B. Plenio, and I. A. Walmsley, *Phys. Rev. Lett.* **108**, 060502 (2012); T. J. Bartley, P. J. D. Crowley, A. Datta, J. Nunn, L. Zhang, and I. Walmsley, *Phys. Rev. A* **87**, 022313 (2013).
- [11] T. Opatrný, G. Kurizki, and D.-G. Welsch, *Phys. Rev. A* **61**, 032302 (2000).
- [12] S. Olivares, M. G. A. Paris, R. Bonifacio, *Phys. Rev. A* **67**, 032314 (2003).
- [13] J. Wenger, R. Tualle-Brouiri, and P. Grangier, *Phys. Rev. Lett.* **92**, 153601 (2004).
- [14] A. Franzen, B. Hage, J. DiGuglielmo, J. Fiurášek and R. Schnabel, *Phys. Rev. Lett.* **97**, 150505 (2006).
- [15] R. Dong, M. Lassen, J. Heersink, C. Marquardt, R. Filip, G. Leuchs and U. L. Andersen, *Nature Physics* **4**, 919 (2008).
- [16] A. Ourjoumtsev, F. Ferreyrol, R. Tualle-Brouiri, and P. Grangier, *Nature Phys.* **5**, 189 (2009).
- [17] H. Takahashi, J. S. Neergaard-Nielsen, M. Takeuchi, M. Takeoka, K. Hayasaka, A. Furusawa, and M. Sasaki, *Nature Photonics* **4**, 178 (2010).
- [18] Y. Kurochkin, A. S. Prasad, and A. I. Lvovsky, *Phys. Rev. Lett.* **112**, 070402 (2014).
- [19] M. Aspelmeyer, S. Gröblacher, K. Hammerer, and N. Kiesel, *J. Opt. Soc. Am. B* **27**, A189 (2010).
- [20] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, *Rev. Mod. Phys.* **86**, 1391 (2014).
- [21] S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, *Phys. Rev. Lett.* **88**, 120401 (2002).
- [22] S. Pirandola, D. Vitali, P. Tombesi, and S. Lloyd, *Phys. Rev. Lett.* **97**, 150403 (2006).
- [23] D. E. Chang, C. A. Regal, S. B. Papp, D. J. Wilson, J. Ye, O. Painter, H. J. Kimble, and P. Zoller, *Proc. Natl. Acad. Sci. U.S.A.* **107**, 1005 (2010).
- [24] M. Asjad, S. Zippilli, P. Tombesi, and D. Vitali, *arXiv:1411.7216* (2014).
- [25] C. K. Law, *Phys. Rev. A* **51**, 2537 (1995).
- [26] S. Bose, K. Jacobs, and P. L. Knight, *Phys. Rev. A* **59**, 3204 (1999).
- [27] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, *Phys. Rev. Lett.* **91**, 159903 (2003).
- [28] J. Li, S. Gröblacher, and M. Paternostro, *New J. Phys.* **15**, 033023 (2013).
- [29] A. Nunnenkamp, K. Børkje, and S. M. Girvin, *Phys. Rev. Lett.* **107**, 063602 (2011).
- [30] K. Stannigel, P. Komar, S. J. M. Habraken, S. D. Bennett, M. D. Lukin, P. Zoller, and P. Rabl, *Phys. Rev. Lett.* **109**, 013603 (2012).
- [31] Bing He, *Phys. Rev. A* **85**, 063820 (2012).
- [32] J.-Q. Liao, H. K. Cheung, and C. K. Law, *Phys. Rev. A* **85**, 025803 (2012).
- [33] Xun-Wei Xu, Yuan-Jie Li, and Yu-xi Liu, *Phys. Rev. A* **87**, 025803 (2013).
- [34] A. Kronwald, M. Ludwig, and F. Marquardt, *Phys. Rev. A* **87**, 013847 (2013).
- [35] Xun-Wei Xu and Yuan-Jie Li, *J. Phys. B: At. Mol. Opt. Phys.* **46**, 035502 (2013).
- [36] U. Akram, W. P. Bowen, and G. J. Milburn, *New J. Phys.* **15**, 093007 (2013).
- [37] J.-Q. Liao and F. Nori, *Scientific Reports* **4**, 6302 (2014).
- [38] A. H. Safavi-Naeini, T. P. Mayer Alegre, J. Chan, M. Eichenfield, M. Winger, Q. Lin, J. T. Hill, D. E. Chang, and O. Painter, *Nature* **472**, 69 (2011).
- [39] J. Chan, T. P. Mayer Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Gröblacher, M. Aspelmeyer, and O. Painter, *Nature (London)* **478**, 89 (2011).
- [40] J. D. Teufel, T. Donner, D. Li, J. H. Harlow, M. S. Allman, K. Cicak, A. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, *Nature* **475**, 359 (2011).
- [41] K. W. Murch, K. L. Moore, S. Gupta, and D. M. Stamper-Kurn, *Nature Phys.* **4**, 561 (2008).
- [42] A. Xuereb, C. Genes and A. Dantan, *Phys. Rev. Lett.* **109**, 223601 (2012).
- [43] H. Kaviani, C. Healey, M. Wu, R. Ghobadi, A. Hryciw, and P. E. Barclay, *arXiv:1412.4431* (2014).
- [44] M. R. Vanner, J. Hofer, G. D. Cole, and M. Aspelmeyer, *Nature Communications* **4**, 2295 (2013).
- [45] P. Sekatski, M. Aspelmeyer, and N. Sangouard, *Phys. Rev. Lett.* **112**, 080502 (2014).
- [46] M. R. Vanner, I. Pikovski, G. D. Cole, M. S. Kim, Č. Brukner, K. Hammerer, G. J. Milburn, and M. Aspelmeyer, *Proc. Natl. Acad. Sci. U.S.A.* **108**, 16182 (2011).
- [47] B. P. Abbott *et al.*, *Rep. Prog. Phys.* **72**, 076901 (2009).
- [48] A. S. Parkins and H. J. Kimble, *Phys. Rev. A* **61**, 052104 (2000).
- [49] N. Kiesel, F. Blaser, U. Delić, D. Grass, R. Kaltenbaek, and M. Aspelmeyer, *Proc. Natl. Acad. Sci. U.S.A.* **110**,

- 14180 (2013).
- [50] J. Millen, P. Z. G. Fonseca, T. Mavrogordatos, T. S. Monteiro, and P. F. Barker, arXiv:1407.3595 (2014).
  - [51] S. Machnes, J. Cerrillo, M. Aspelmeyer, W. Wieczorek, M. B. Plenio, and A. Retzker, Phys. Rev. Lett. **108**, 153601 (2012).
  - [52] S. Bose, K. Jacobs, and P. L. Knight, Phys. Rev. A **56**, 4175 (1997).
  - [53] S. Mancini, V. I. Man'ko, and P. Tombesi, Phys. Rev. A **55**, 3042 (1997).
  - [54] A. Schliesser, R. Rivière, G. Anetsberger, O. Arcizet, and T. J. Kippenberg, Nature Physics **4**, 415 (2008).
  - [55] W. Son, M. S. Kim, J. Lee, and D. Ahn, J. Mod. Opt. **49**, 1739 (2002); M. S. Kim, G. Antesberger, C. T. Bodendorf, and H. Walther, Phys. Rev. A **58**, R65(R) (1998); G. S. Agarwal, Quantum Optics, Cambridge University Press (2012).
  - [56] A. Ferraro, S. Olivares, and M. G. A. Paris, *Gaussian states in continuous variable quantum information* (Bibliopolis, Napoli, 2005); arXiv:quant-ph/0503237.
  - [57] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 565 (2009).
  - [58] J. Lee, M. S. Kim, Y. J. Park, and S. Lee, J. Mod. Opt. **47**, 2151 (2000).
  - [59] G. Vidal and R. F. Werner, Phys. Rev. A **65**, 032314 (2002).
  - [60] K. Jacobs, P. Tombesi, M. J. Collett, and D. F. Walls, Phys. Rev. A **49**, 1961 (1994).
  - [61] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
  - [62] E. Verhagen, S. Deléglise, S. Weis, A. Schliesser, and T. J. Kippenberg, Nature **482**, 63 (2012).
  - [63] N. Kiesel, private communication.
  - [64] P. Barker, private communication.
  - [65] E. T. Campbell and J. Eisert, Phys. Rev. Lett. **108**, 020501 (2012).
  - [66] N. C. Menicucci, P. van Loock, M. Gu, C. Weedbrook, T. C. Ralph, M. A. Nielsen, Phys. Rev. Lett. **97**, 110501 (2006).